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Supplementary Materials for

Using the Earth as a Polarized Electron Source to Search for Long-Range Spin-Spin Interactions

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Figs. S1 to S3
Table S1
References (*45*–*50*)

Supplementary Materials: Figures S1, S2 and S3 and Table S1.

Fig. S1. The average mineral density as a function of depth in the PREM model (*30*) and as calculated from our model.

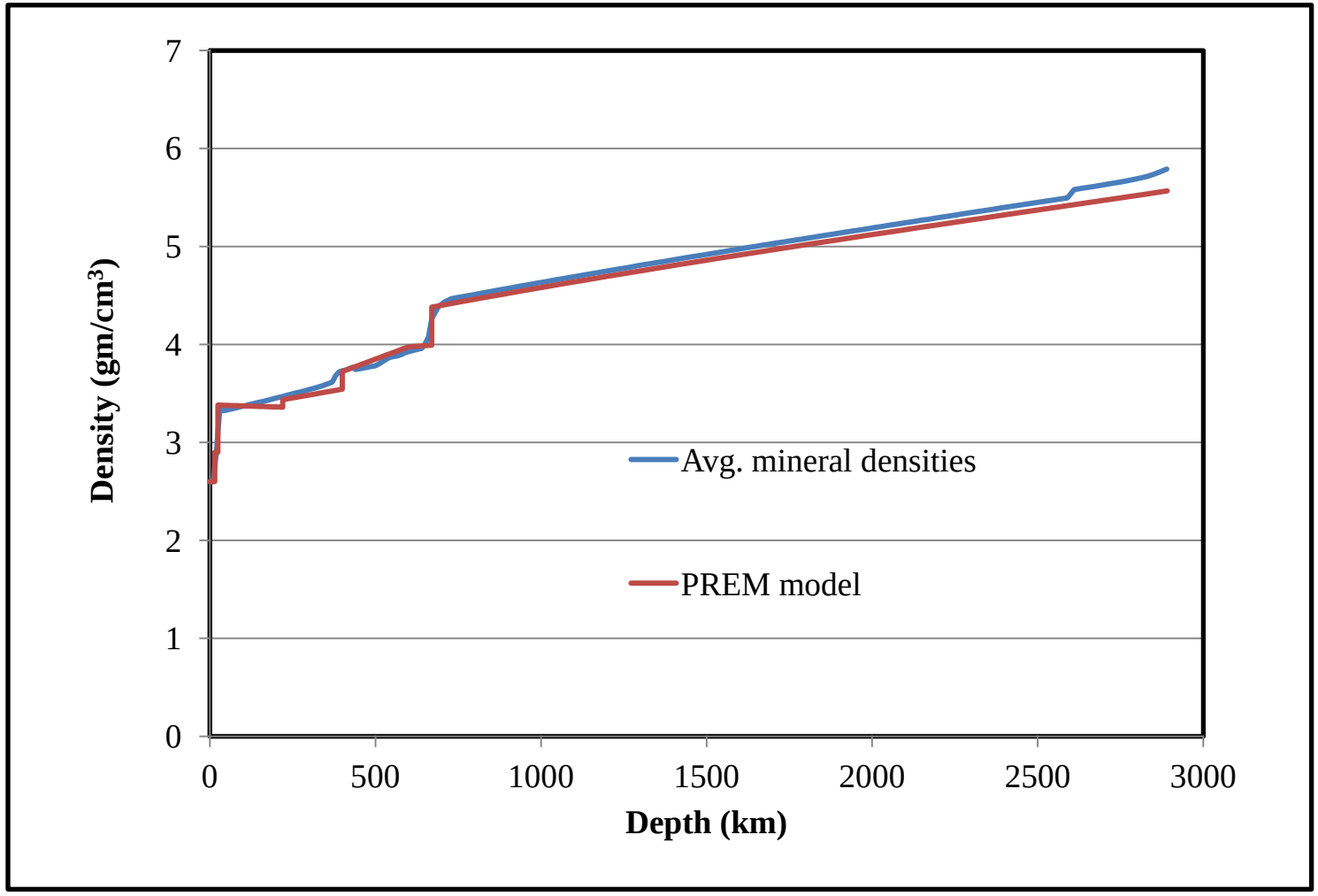
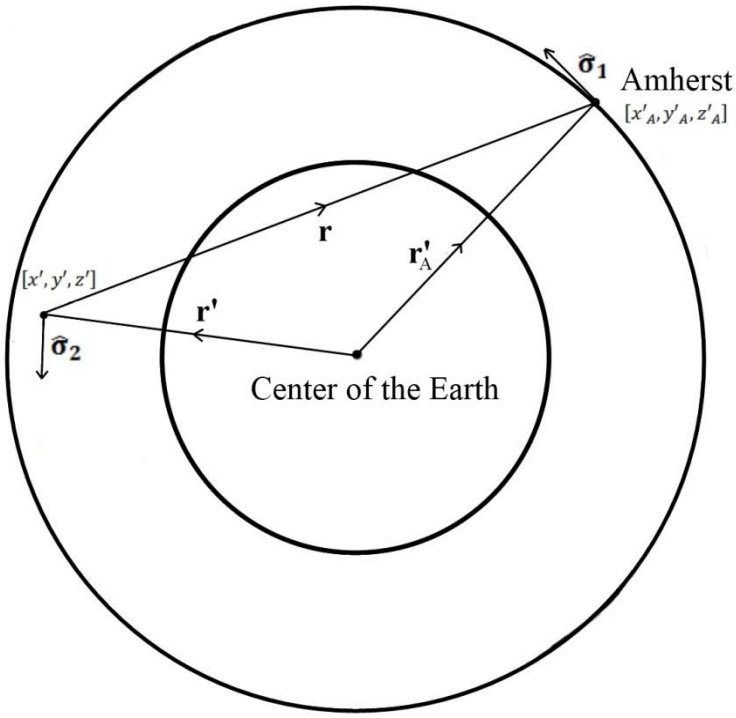


Fig. S2. Details of the integration of the potential over the Earth volume. The integration is over all of the volume from the core-mantle boundary (R_{CM}) to the surface (R_S). The potential (either given by Eq. 1, 2 or 3) is evaluated at $\mathbf{r} = \mathbf{r'_A} - \mathbf{r'}$ where each point in the Earth is described by the vector $\mathbf{r'}$ and the location of Amherst (or Seattle) is designated by the vector $\mathbf{r'_A}$. The geoelectron spin direction $\hat{\boldsymbol{\sigma}}_2$ is assumed to be antiparallel to the Earth magnetic field **B** at $\mathbf{r'}$. The unit vector $\hat{\boldsymbol{\sigma}}_1$ is the spin-sensitive direction of the apparatus in Amherst (or Seattle). The unpaired electron density, $\rho(r')$ is given by the "Equivalent Fe^{2+} e^-" density shown in Fig. 1. The temperature profile $T(r')$ is taken from (*29*) and (*30*). To establish the bounds on the coupling coefficients we require V_{total} to be less than the energy bound established on the spin coupling energy (β) in the spin-sensitive direction by the various experiments.



$\hat{\boldsymbol{\sigma}}_1$ = spin sensitive direction of apparatus at Amherst

$\hat{\boldsymbol{\sigma}}_2 = -\hat{\mathbf{B}}$

$\mathbf{r'_A}$ = Radial position vector of Amherst = $[x'_A, y'_A, z'_A]$

$\mathbf{r'}$ = Radial position vector of electron = $[x', y', z']$

$x' = r' \sin\theta' \sin\varphi'$

$y' = r' \sin\theta' \cos\varphi'$

$z' = r' \cos\theta'$

$$\mathbf{r}(r', \theta', \varphi') = \mathbf{r'_A} - \mathbf{r'} = [(x'_A - x'), (y'_A - y'), (z'_A - z')]$$

$$V_{total} = \int_0^{2\pi} \int_0^{\pi} \int_{R_{CM}}^{R_S} r'^2 \sin\theta' * \rho(r') * \frac{2\mu_b B}{k_b T(r')} * V(\mathbf{r}(r', \theta', \varphi'))\, dr' d\theta' d\varphi'$$

Fig. S3. Contributions to the integration from 5 km thick spherical shells of various depths for $1/r$ (red) and $1/r^2$ (blue) potentials.

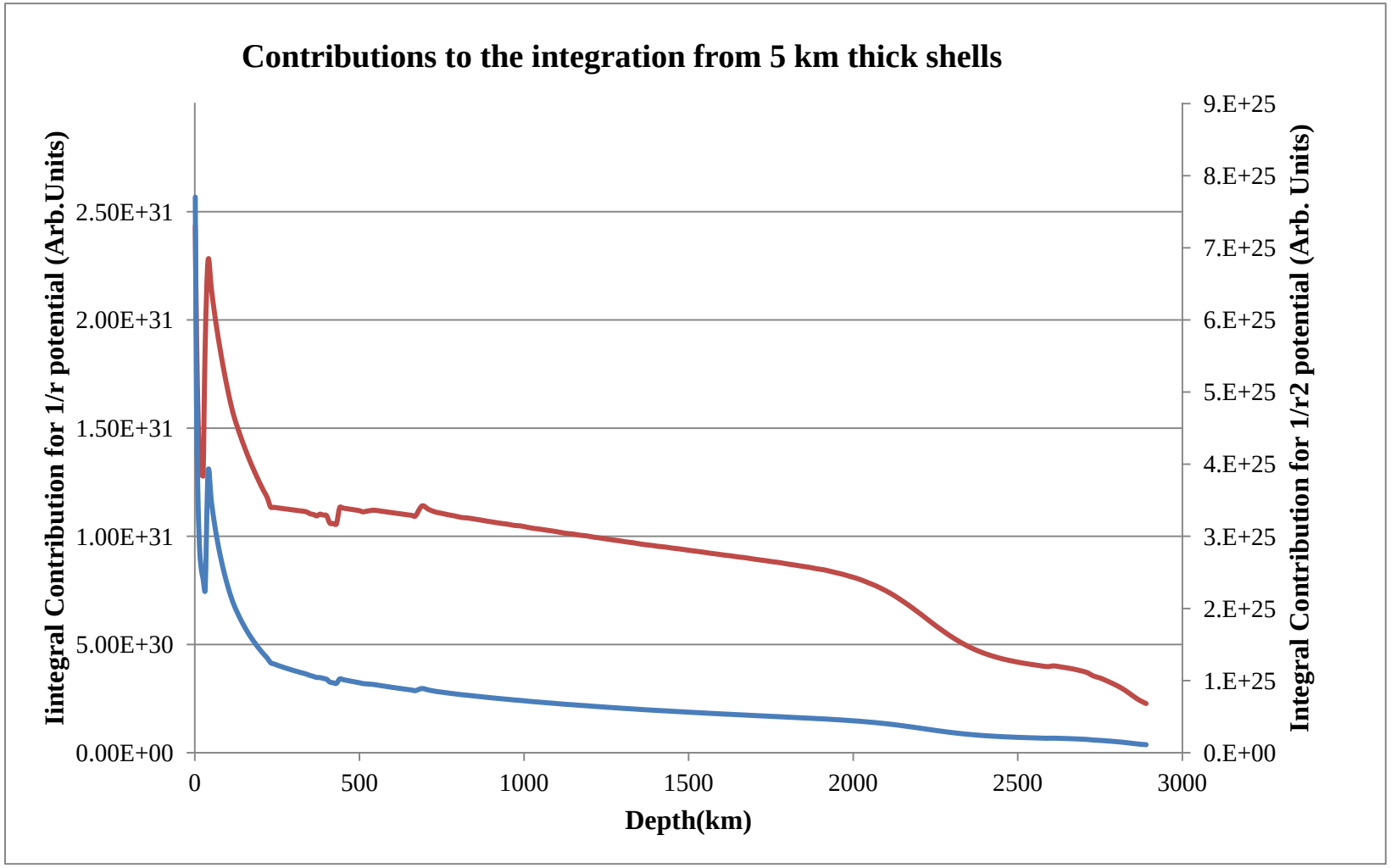


Table S1. Thermoelastic parameters and iron fractions used in the model (*23, 24, 27, 28, 29, 33, 44-50*). Most of the parameters were chosen from experimental results on iron-bearing minerals with compositions as close to the pyrolite model as possible. Thermal expansion coefficients at high temperatures are calculated using the equation: $\alpha=\alpha_0+\alpha_1 T+\alpha_2/T^2$. Asterisks indicate no data available. The volume fractions are approximate as the actual volumes used are depth dependent as indicated in Table 1. For some minerals, the iron fractions are slightly adjusted with changes in depth in order to maintain the pyrolite approximation. Mineral abbreviations: Olivine (Ol), Garnet (Gr), Clinopyroxene (Cpx), Orthopyroxene (Opx), Majorite (Mj), Wadsleyite (Wd), Ringwoodite (Rw), Ferropericlase (Fp), Perovskite (Pv), Post-Perovskite (PPv).

Min.	Vol. (%)	Fe/(Fe+Mg) %	Density (g/cm^3)	K_0 (GPa)	dK/dP	dK/dT (GPa/K)	α_0 (10^{-5})	α_1 (10^{-8})	α_2
Ol	60	0.1	3.34	130	4.2	-0.0164	2.7	2.2	*
Gr	16	0.2	3.72	168	4.4	-0.0185	2.31	5.96	-0.4538
Cpx	18	0.11	3.32	117	4.2	-0.013	3.18	*	*
Opx	6	0.09	3.28	116	7.8	-0.0238	2.97	0.57	*
Mj	43	0.11	3.64	166	4.2	-0.014	2.5	*	*
Wd	57	0.1	3.60	171	4.7	-0.0175	3.4	*	*
Rw	57	0.1	3.68	188	4.1	-0.021	3.0	*	*
HS Fp	18	0.12	3.86	162	4.0	-0.017	3.76	*	*
Mg-Pv	75	0.1	4.21	255	4.0	-0.021	1.7	*	*
Ca-Pv	7	0	4.23	237	4.0	-0.036	3.22	0.688	
PPv	75	0.1	4.19	223	4.0	-0.015	1.48	*	*

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